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Honors Physics
Period A

Lab Experiment #10. Simple Harmonic Motion of Spring and Pendulum

Purpose: To demonstrate using a spring and a pendulum how the parameters of simple harmonic motion be calculated.

Materials: None.

Equipment: Stopwatch, spring stand and clamp, C-clamp, spring, string with looped ends, 0.5 kg and 1 kg masses, graduated metric ruler.

Introduction:

Simple harmonic motion is a periodic vibration about an equilibrium position that repeats itself at standard periods. A restoring force is present and proportional to the distance from equilibrium (its amplitude), which restores motion to equilibrium and then displaces it proportionally in the opposite direction. Speed reaches maximum velocity at equilibrium position, and force and acceleration reach their maximum at maximum displacement from equilibrium. Simple harmonic motion can easily be calculated and predicted using mathematical formulas, and this experiment seeks to demonstrate two instances of simple harmonic motion using a spring and a pendulum.

A mass attached to a spring that oscillates around an equilibrium position is an example of simple harmonic motion. The period of vibration of a mass held by a spring can be derived using the formula $T = 2\pi\sqrt{\frac{m}{k}}$ where m is the mass and k is the force constant of the sprint. The masses used in the experiment are known, so they will not have to be measured. We can employ Hooke's Law to find the force constant of the string, which states that the change in force is equal to the negative of the spring constant times the change in spring length. This change in force is equal to the weight of the mass in newtons, and the change in distance will be measured using a ruler. Upon finding the force constant, we can determine the period of vibration, which we can double-check by timing the motion of the spring using a stopwatch (counting 20 intervals and dividing, for better accuracy). We will also ensure that changing the amplitude of the motion does not affect the results, as the equations indicate it shouldn't.

Another example of simple harmonic motion is a pendulum. The vibration time of a pendulum can be calculated using the equation $T = 2\pi\sqrt{\frac{L}{g}}$ where L is equal to the length of the pendulum (measured to the center of gravity of the bob) and g is acceleration due to gravity (accepted to be 9.80 m/s^2). In this experiment, we will attempt to solve for g using the equation $g = \frac{4\pi^2 L}{T^2}$ where T is the period of vibration. By comparing this to the accepted value of g, we can determine if the formula accurately predicts simple harmonic motion. Additionally, we can ensure that the vibration time is proportional to the length of the pendulum, but is unaffected by the mass, as the equations indicate. To do this, we will test different masses at two equivalent lengths to see if their time is equal.

Lab A: Spring-Mass Experiment

Procedure:

1. The required equipment and materials were retrieved from their locations in the lab.
2. The stand was attached to the edge of the table using the C-clamp.
3. One end of the spring was hooked on to the clamp attached to the stand.
4. The relaxed length of the spring was measured using the ruler.
5. The 0.5 kg mass was hooked on to the bottom of the spring and steadied. The stretched length of the spring was then measured.
6. The spring was stretched about 1 cm from equilibrium and released, to create a small amplitude vibration. Twenty vibration cycles were timed using the stopwatch.
7. Step 6 was repeated with a larger amplitude vibration, about 5 cm from equilibrium.
8. Steps 5-7 were repeated again with the 1 kg mass in place of the 0.5 kg mass, and once more with both the 0.5 kg and 1 kg masses hooked together.
9. The mass was detached, and the spring was removed from the hook. All items were returned to their proper locations in the lab.

Observations:

Measurements

The following measurements were made during each trial of the experiment:

Original Length (± .005 m)	.261		
Mass (kg)	**Final Length (± .005 m)**	**Small Amplitude T_{20} (± .3 s)**	**Large Amplitude T_{20} (± .3 s)**
.500	.407	15.5	15.5
1.000	.556	21.9	21.7
1.500	.703	26.5	26.6

Results:

Mass Weight

In order to calculate the spring constant, we must find the change in force caused by the mass, or ΔF. The weight of each mass can be calculated using the following equation:

$$\textbf{Weight}\ (\Delta F) = mass \times acceleration(gravity)$$

The mass of the weights are exact, and acceleration due to gravity is assumed to be 9.80 m/s^2.

Mass (kg)	Weight, ΔF (N)
.500	.500 × 9.80 = **4.90**
1.000	1.000 × 9.80 = **9.80**
1.500	1.500 × 9.80 = **14.7**

Change in Spring Length

To find the spring constant, we must also calculate the change in the length of the spring, or Δx. This can be calculated by subtracting the relaxed 'original' length of the spring from its stretched 'final' length.

Mass (kg)	Original Length (± .005 m)	Final Length (± .005 m)	Length Delta, Δx (± .01 m)
.500	.261	.407	.15 ± 3.1%
1.000		.556	.30 ± 2.8%
1.500		.703	.44 ± 2.6%

Force Constant

The force constant of the spring (measured in Newtons/meters) can be found using the equation:

$$\textbf{Force constant (k)} = \text{change in force } (\Delta F) \text{ / change in length } (\Delta x)$$

Mass (kg)	Weight, ΔF (N)	Length Delta, Δx (± .01 m)	Force Constant, k (N/m)
.500	4.90	.15 ± 3.1%	33 ± 1
1.000	9.80	.30 ± 2.8%	33 ± 1
1.500	14.7	.44 ± 2.6%	33 ± 1
		$\mathbf{k}_{average}$	33 ± 3.0%

Vibration Period (Predicted)

Using the mass and average force constant, we can calculate the predicted vibration period of each trial. This can be calculated using the following equation:

$$\textbf{Period } (T) = 2\pi\sqrt{\frac{\text{mass } (m)}{\text{force constant } (k)}}$$

Mass (kg)	Predicted Vibration Period (s)
.500	$2\pi\sqrt{\frac{.500}{33 \pm 3.0\%}}$ = **.77 ± .01 (1.5%)**
1.000	$2\pi\sqrt{\frac{1.000}{33 \pm 3.0\%}}$ = **1.1 ± .02 (1.5%)**
1.500	$2\pi\sqrt{\frac{1.500}{33 \pm 3.0\%}}$ = **1.3 ± .02 (1.5%)**

Observed Vibration Period

By taking the average measured time for the vibrations of each mass and dividing by 20, we can find the observed time for one period of vibration.

Mass (kg)	Small Amplitude T_{20} (± .3 s)	Large Amplitude T_{20} (± .3 s)	Average T_{20} (± .6 s)	Observed Vibration Period (s)
.500	15.5	15.5	15.5	.78 ± .03 (3.9%)
1.000	21.9	21.7	21.8	1.09 ± .03 (2.8%)
1.500	26.5	26.6	26.6	1.33 ± .03 (2.3%)

Percent Difference

We can calculate a percent difference between the predicted and observed vibration periods using the equation:

$$\%\,error = \left|\frac{predicted - observed}{smaller\ quantity}\right| \times 100$$

Mass (kg)	Predicted Vibration Period (s)	Observed Vibration Period (s)	Percent Difference
.500	.77 ± .01 (1.5%)	.78 ± .03 (3.9%)	1.3%
1.000	1.1 ± .02 (1.5%)	1.09 ± .03 (2.8%)	0.9%
1.500	1.3 ± .02 (1.5%)	1.33 ± .03 (2.3%)	2.3%

Discussion:

As the results indicate, the predicted and observed results for the period of vibration were within the limits of measurement uncertainty, showing that the equation to calculate the period of vibration is consistent with real-life measurements. The results also confirm that the period of vibration is not noticeably affected by different amplitudes of vibration, as the time measurements for each amplitude were nearly equal.

There was the potential for several sources of error in the experiment. Aside from time or measurement error using the ruler and stopwatch, had the vibration not been set up correctly, it would have impact the results. Such error may include the stand being unstable or the masses moving too far during the vibration period. In addition, an incorrect assumption of the acceleration due to gravity or an incorrect measurement of the weight of the masses may have impacted the results. However, these sources of error do not seem to have affected the results.

Conclusion:

The results of the experiment support the theory of simple harmonic motion used to derive the formula $T = 2\pi\sqrt{\frac{m}{k}}$.

Lab B: Pendulum Experiment

Procedure:

1. The required equipment and materials were retrieved from their locations in the lab.
2. The stand was attached to the edge of the table using the C-clamp.
3. One end of the string was hooked on to the clamp attached to the stand.
4. The 0.5 kg mass was hooked on to the bottom of the string and steadied. The length of the string to the center of gravity of the mass was measured.
5. The pendulum was set into motion. Twenty vibration cycles were timed using the stopwatch.
6. Steps 4-5 were repeated with the 1 kg mass.
7. The mass was removed from the string. The loose end of the string was then attached to the clamp, shortening it by half.
8. Steps 4-6 were repeated with the shorter string.
9. The mass was detached, and the string was removed from the hook. All items were returned to their proper locations in the lab.

Observations:

Measurements

The following measurements were made during each trial of the experiment:

Mass (kg)	Full String		Half String	
	String Length (± .005 m)	T_{20} (± .3 s)	String Length (± .005 m)	T_{20} (± .3 s)
.500	.841	35.1	.434	26.3
1.000	.853	37.4	.445	26.8

Results:

Observed Vibration Period

By dividing each period of time by 20, we can find the observed time for one period of vibration.

Mass (kg)	T_{20} (± .3 s)	Observed Vibration Period (s)
Full String		
.500	35.1 ± 0.9%	1.8 ± 0.9%
1.000	37.4 ± 0.8%	1.9 ± 0.8%
Half String		
.500	26.3 ± 1.1%	1.3 ± 1.1%
1.00	26.8 ± 1.1%	1.3 ± 1.1%

Measure of Gravity

Part of the experiment to test the theories of simple harmonic motion using a pendulum includes calculating the value of gravity. Using the length of the pendulum and its vibration period, we can determine the acceleration due to gravity by using the following formula:

$$\textbf{acceleration of gravity } (g) = \frac{4\pi^2 \times \text{length } (L)}{\text{period}^2 \ (T^2)}$$

We can find an approximation of acceleration due to gravity in the experiment by using the preceding formula to calculate gravity for each length, and then averaging the values together.

Mass (kg)	String Length	Vibration Period (s)	Calculated Gravity g (m/s^2)
		Full String	
.500	.841 ± 0.6%	1.8 ± 0.9%	$\frac{4\pi^2 \times .841 \pm 0.6\%}{(1.8 \pm 0.9\%)^2}$ = **10.2 ± 2.4%**
1.000	.853 ± 0.6%	1.9 ± 0.8%	$\frac{4\pi^2 \times .853 \pm 0.6\%}{(1.9 \pm 0.8\%)^2}$ = **9.3 ± 2.2%**
		Half String	
.500	.434 ± 1.2%	1.3 ± 1.1%	$\frac{4\pi^2 \times .434 \pm 1.2\%}{(1.3 \pm 1.1\%)^2}$ = **10.1 ± 3.4%**
1.00	.445 ± 1.1%	1.3 ± 1.1%	$\frac{4\pi^2 \times .445 \pm 1.1\%}{(1.3 \pm 1.1\%)^2}$ = **10.4 ± 3.3%**
		Average g (m/s^2)	10.0 ± 11.3%

Percent Difference

We can calculate a percent difference between the calculated value of *g* and the assumed value of *g*, 9.80 m/s^2, using the equation:

$$\%\,error = \left|\frac{\text{calculated } g - 9.80\ m/s^2}{9.80\ m/s^2}\right| \times 100$$

Average Calculated g (m/s^2)	Actual g (m/s^2)	Percent Difference
10.0	9.80	2.0%

Discussion:

Our average value of *g*, while not completely accurate, is not far from the accepted value and easily falls within the range of uncertainty. The vibration times of the pendulum were affected by its length, but were mostly unaffected by the weight of the mass.

There was the potential for several sources of error in the experiment, which would explain why our average value of *g* did not meet the accepted value. If the pendulum was not properly set in motion, it would have led to incorrect measurements. As we could not determine the exact center of gravity, the measurement of the length of the pendulum may not have been accurate (though still covered by the range of uncertainty). Additional sources of error include timing the cycles of the pendulum, which had to be done by eyesight, and whether the mass was centered when the string was halved.

Conclusion:

Within the margin of uncertainty, the results of the experiment support the theory of pendulum simple harmonic motion that is used to derive the formula $T = 2\pi\sqrt{\frac{L}{g}}$.